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# ARMAMENT RESEARCH AND DEVELOPMENT ESTABLISHMENT

## BASIC RESEARCH AND WEAPON SYSTEMS ANALYSIS DIVISION

## A.R.D.E. REPORT (B) 27/57

**The ultimate distribution of energy in a spherical explosion**

C. K. Thornhill



[redacted]

Fort Halstead,
Kent.

October
1957

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Ministry of Supply

ARMAMENT RESEARCH AND DEVELOPMENT ESTABLISHMENT

A.R.D.E. REPORT (B) 27/57

# The ultimate distribution of energy in a spherical explosion

C. K. Thornhill (B.1)

Summary

Ultimately, at relatively large distances from the source, there is some similarity between the blast waves from all explosions. Various authors have demonstrated this mathematically, but have not drawn attention to the fact that their results are not entirely compatible with the classical hypothesis that, ultimately, blast waves become less and less aware of the nature of the explosion and tend to depend only on the magnitude of the energy release.

The same hypothesis has been used to derive atmospheric scaling laws, which therefore need alternative interpretation if the hypothesis is found to be false.

The ultimate flow behind a decaying spherical shock is studied here from this point of view. It is concluded that the classical hypothesis is false, and is not essential to the mathematical similarity. It has been shown elsewhere ~~[10]~~ * that it is also not essential to the atmospheric scaling rules.

The theory developed in making these conclusions suggests that the flow behind a spherical shock is divided into two parts by an inner sphere, such that the flow within this sphere has no influence on the decay of the shock. The excess energy in the outer part increases indefinitely in proportion to the shock radius, and that within the inner sphere must therefore correspondingly decrease indefinitely since the total excess energy must remain finite and equal to the energy released by the explosion.

A hypothesis is suggested, in an attempt to obtain a higher order asymptotic solution, for the blast wave, than the well-known form given by previous authors. This solution is compared fully with experimental observations.

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## Contents



## 1. Introduction

It has long been known that, far away from the origin of explosion, there is ultimately some similarity between the blast waves from all explosions, whatever the source and nature of the energy release which originally initiated them. Bethe [1] first showed that, if R is the radius of a decaying spherical shock, then, as $R \to \infty$, the peak-overpressure behind the shock ultimately behaves as $[R \log^{\frac{1}{2}} R]^{-1}$, the duration of positive overpressure behaves as $\log^{\frac{1}{2}} R$, and the impulse per unit area in the positive overpressure phase behaves as $R^{-1}$. Kirkwood and Brinkley [2] developed a theory covering the whole course of an explosion, based on a physical assumption about energy, which led to the same asymptotic form for large values of R. Later, Whitham [3] re-derived Bethe's results by an alternative approach, aimed, like Bethe's method, at correcting the imperfections of the 'acoustic' solution of the problem of a decaying shock.

Both Bethe and Kirkwood and Brinkley define the energy of the shock wave, as it crosses the sphere of radius R, as the nett work done on the undisturbed atmosphere exterior to the sphere R. But only by making assumptions about the motion in the negative phase of the blast wave, and thus assessing effectively the final nett amount of work done on the undisturbed atmosphere external to the sphere of radius R, do they reach the conclusion that this nett energy tends ultimately to zero as $\log^{-\frac{1}{2}} R$ when R is large, thus indicating a very slow final dissipation of energy, a result first derived by Penney [11]. None of these authors has, however, drawn attention to the difficulties associated with considerations of the immediate energy distribution <u>behind</u> a decaying spherical shock. If, instead of considering the total nett work done on the undisturbed atmosphere external to the sphere

R, the development of this energy with time is considered, it is found, at once, without any assumptions concerning the negative phase, that the work done, up to the end of the positive overpressure phase, on the atmosphere originally outside the sphere R, behaves as R when R is large, and thus increases indefinitely as $\alpha$ increases. This is a curious result, and, despite the fact that such a curious energy situation must be restored by a negative phase, it is of the greatest importance to an understanding of explosion phenomena (particularly as applied to target response) to examine the consequences of such an anomalous energy distribution. Whitham [3] does not deal with energy considerations, but in a later paper [12] he mentions a similar anomaly, namely that the mass flowing from inside the sphere R to outside the sphere R during the positive overpressure phase ultimately behaves as R, and thus necessitates the existence of a negative phase in which it can return. The anomaly does not exist in plane one-dimensional flow; in this case the excess energy in the positive overpressure phase tends to a finite quantity as R increases. In axially symmetric flow, the excess energy in the positive overpressure phase behaves ultimately as $R^{\frac{1}{2}}$ when R is large.

In the absence of any real attempt to face up to the difficulties of a positive overpressure phase containing an indefinitely increasing amount of energy, continuing appeal has tacitly been made to the classical hypothesis of explosion theory, namely that, for large R, the motion behind a blast shock becomes dependent dominantly on a finite quantity of energy associated with the energy released by the explosion, and less and less on the nature and mode of the energy release. The asymptotic results described above would only be compatible with such a hypothesis if the energy in the positive overpressure phase tended to a finite non-zero amount for large R, since, as will be shown in this paper, the decay of a spherical shock is determined entirely by the motion in a limited region of flow just behind the shock, not much different from the positive overpressure phase. The hypothesis is therefore now believed to be false.

This same classical hypothesis has been used by Sachs [13] to derive atmospheric scaling laws, though the same scaling laws were derived previously by Taylor [14], close in to a very intense explosion [cf. 10] on the different hypothesis that the total excess energy within the spherical shock is conserved. The present author [10] has shown that Sachs' rule for atmospheric scaling can be derived, and generalised, without appeal to the classical energy hypothesis, and that its practical usefulness is not therefore nullified by the falsity of the hypothesis. Instead, Sachs' rule is seen to be not an asymptotic law, as first derived, but a rule for the 'middle-distance' which is of far greater importance in practical applications. Taylor's derivation of the atmospheric scaling rule, close in to a very intense explosion, also needs further consideration, since the high temperatures associated with the high energy release per unit volume of a very intense explosion imply a high rate of loss of energy from the system within the spherical shock by thermal radiation [cf. 15]. The derived atmospheric scaling rules may therefore only be expected to apply approximately, under the conditions of Taylor's similarity solution, over ranges of R in which the energy lost to the system by thermal radiation is either small compared with the total energy of the system, or scales consistently with the derived scaling rules.

In the present paper, the equations of unsteady non-homentropic compressible flow with spherical symmetry are reduced to characteristic form (section 2), in terms of variables which are effectively the Riemann functions for the corresponding plane flow. The boundary conditions at a general spherical shock front are derived in the same variables (section 3) and an iterative solution is developed. This method of solution was developed independently, but was found later to be similar to that used by Bethe [1], although the present solution is carried further than Bethe's solution. Energy and impulse considerations are introduced (section 4), to derive the limitations on the form of the shock decay, and the results derived are in accord with previous results described above, but are taken to higher order approximations. The existence of a limiting characteristic or wave-front, behind the leading shock, is demonstrated (section 5), as was also done previously by Whitham, namely the first positive characteristic which fails to overtake the shock in finite distance. This marks the rear of what is defined here as the shock decay phase of the motion, for it divides the flow behind the shock into two parts, a spherical annulus (between the shock and the limiting characteristic) which alone controls the decay of the shock, and, inside it, a spherical region which cannot influence the shock decay process. The previous results (of section 4) then show that, although the total excess energy within the leading shock is always finite and equal to the energy released by the explosion (less possible radiation losses), its components, in the two regions defined above, respectively increase and decrease indefinitely in proportion to R, when R is large, and thus contradict the classical hypothesis that the flow in the shock decay phase immediately behind the leading shock is determined dominantly by a finite quantity of energy.

The difficulties of obtaining a higher order approximation, without recourse to hypothesis, or to the flow conditions behind the shock decay phase, are introduced (section 6), and a hypothesis for a higher order approximation is suggested. On such a hypothesis, the ultimate form of the pressure-pulse in a blast wave is derived (section 7); the higher order results from the hypothesis are compared with experimental observations (section 8), and show promising agreement. Nevertheless, even if these higher order results are ultimately found to be sound, they do not apply, with practical accuracy, closer in than about 20 ft. from a one-pound charge of conventional explosive at sea-level, where the shock peak overpressure is about one-sixth of an atmosphere. It still remains true, therefore, as stated by Bethe [1], that "there is considerable danger in using these relations for moderate pressures where the pulse has not yet reached its limiting form".

The real practical problem of explosion theory therefore remains, namely to provide a consistent and comprehensive working solution for the 'middle-distance' of moderate peak overpressures. For it is in the 'middle-distance' [cf. 16] that the critical conditions of target response generally occur.

This paper, then, makes no contribution to the really important practical problem, except in so far as it seeks to improve understanding of the ultimate asymptotic form of a blast wave, which must, of necessity, be an end-condition for the 'middle-distance' problem. It is clear that a proper understanding of the end-condition is a necessary preliminary, in any case. It may, or may not, provide the key also to a useful approach to the more practical problem.

## 2. Unsteady non-homentropic flow with spherical symmetry

The equations of motion of the air behind the leading blast shock in a spherical explosion, in regions where no further shocks occur, are:
from the conservation of momentum,

$$u_t + uu_r + p_r/\rho = 0; \tag{2.1}$$

from the conservation of mass,

$$\rho_t + u\rho_r + \rho u_r + 2u\rho/r = 0; \tag{2.2}$$

and from the conservation of energy

$$p_t + up_r = a^2(\rho_t + u\rho_r). \tag{2.3}$$

p denotes pressure; $\rho$, density; a, sound-speed; u, particle velocity; r denotes distance from the centre of spherical symmetry and t, time; suffixes r and t denote partial derivatives; and air is assumed to behave as an ideal polytropic gas, with equation of state

$$\left\{\frac{p}{p_0}\right\} = \left\{\frac{\rho}{\rho_0}\right\}^{\gamma} \exp\left\{\frac{S - S_0}{c_v}\right\}, \tag{2.4}$$

and sound-speed $$a = (\gamma p/\rho)^{\frac{1}{2}}. \tag{2.5}$$

S denotes entropy, $c_v$ specific heat at constant volume; and $\gamma$, the ratio of specific heats at constant pressure and constant volume respectively, namely $c_p/c_v$, is taken to have the value 7/5 for air; suffix o refers here to conditions in the undisturbed uniform atmosphere at rest. In addition

$$c_v = \frac{\mathcal{R}}{\gamma - 1} \tag{2.6}$$

and

$$\frac{p}{\rho} = \mathcal{R}\Theta \tag{2.7}$$

where $\mathcal{R}$ is a constant, and $\Theta$ denotes absolute temperature.

Fundamental units are now chosen, defined by the three quantities, atmospheric pressure $p_0$, atmospheric sound-speed $a_0$, and an arbitrary length L, and the following notation is adopted,

$$\underline{r} = \frac{r}{L}, \quad \underline{u} = \frac{u}{a_0}, \quad \underline{a} = \frac{a}{a_0}\left\{a^2 = \frac{\gamma p}{\rho}\right\}$$

$$\underline{t} = \frac{t a_0}{L} \quad \underline{p} = \frac{p}{p_0} \quad \underline{\rho} = \frac{\rho a_0^2}{p_0} = \frac{\gamma\rho}{\rho_0}$$

i.e. in general $\underline{P} = P/[P]$, where P denotes any physical quantity and [P] is that combination of the fundamental units which has the same dimensions as P. In addition, the convention is adopted of writing $\underline{S} = (S - S_0)/\mathcal{R}$. With this notation and using (2.6), (2.4) and (2.5) may be written

$$\underline{p} = \underline{a}^{2\gamma/(\gamma - 1)}\, e^{-\underline{S}} \tag{2.8}$$

$$\underline{\rho} = \gamma\, \underline{a}^{2/(\gamma - 1)}\, e^{-\underline{S}} \tag{2.9}$$

The equations of motion, (2.1), (2.2) and (2.3), may now be written in the new notation, and $\underline{p}$, $\underline{\rho}$ may be eliminated from them by means of (2.8) and (2.9). The result is

$$\underline{u}_{\underline{t}} + \underline{u}\,\underline{u}_{\underline{r}} + \frac{(1 - \lambda^2)}{\lambda^2}\,\underline{a}\,\underline{a}_{\underline{r}} - \underline{a}^2\,\frac{(1 - \lambda^2)}{(1 + \lambda^2)}\,\underline{S}_{\underline{r}} = 0 \tag{2.10}$$

$$\frac{(1 - \lambda^2)}{\lambda^2}\,\underline{a}_{\underline{t}} + \frac{(1 - \lambda^2)}{\lambda^2}\,\underline{u}\,\underline{a}_{\underline{r}} + \underline{a}\,\underline{u}_{\underline{r}} + \frac{2\,\underline{u}\,\underline{a}}{\underline{r}} = 0 \tag{2.11}$$

$$\underline{S}_{\underline{t}} + \underline{u}\,\underline{S}_{\underline{r}} = 0 \tag{2.12}$$

in which $\lambda^2 = (\gamma - 1)/(\gamma + 1)$.

New dependent variables $\alpha$ and $\beta$ are now chosen instead of $\underline{u}$ and $\underline{a}$, such that

$$\underline{u} = (1 - \lambda^2)(\alpha - \beta), \tag{2.13}$$

$$\underline{a} = 1 + \lambda^2(\alpha + \beta), \tag{2.14}$$

or, alternatively

$$\alpha = \frac{(a - 1)}{2\lambda^2} + \frac{u}{2(1 - \lambda^2)}, \tag{2.15}$$

$$\beta = \frac{(a - 1)}{2\lambda^2} - \frac{u}{2(1 - \lambda^2)}, \tag{2.16}$$

from which it may be seen that $\alpha$ and $\beta$ are, effectively, the Riemann functions which are constant along characteristic lines in unsteady plane homentropic flow. The equations then reduce, after suitable manipulation, to

$$\alpha_{\underline{t}} + [1 + \alpha - (1 - 2\lambda^2)\beta]\,\alpha_{\underline{r}} + \frac{(\alpha-\beta)[1 + \lambda^2(\alpha+\beta)]}{\underline{r}} - \frac{[1 + \lambda^2(\alpha+\beta)]^2}{2(1+\lambda^2)}\,\underline{S}_{\underline{r}} = 0 \tag{2.17}$$

$$\beta_{\underline{t}} - [1 - (1 - 2\lambda^2)\alpha + \beta]\,\beta_{\underline{r}} + \frac{(\alpha-\beta)[1 + \lambda^2(\alpha+\beta)]}{\underline{r}} + \frac{[1 + \lambda^2(\alpha+\beta)]^2}{2(1+\lambda^2)}\,\underline{S}_{\underline{r}} = 0 \tag{2.18}$$

$$\underline{S}_{\underline{t}} + (1 - \lambda^2)(\alpha - \beta)\underline{S}_{\underline{r}} = 0 \tag{2.19}$$

In this form it is seen that the system of equations is hyperbolic, and has three families of characteristic lines in the $(\underline{r}, \underline{t})$-plane. Along the positive characteristic lines,

$$d\underline{r} = [1 + \alpha - (1 - 2\lambda^2)\beta]\,d\underline{t}, \tag{2.20}$$

the intermediate integral relation satisfied is, from (2.17),

$$d\alpha + \frac{(\alpha - \beta)[1 + \lambda^2(\alpha + \beta)]}{[1 + \alpha - (1 - 2\lambda^2)\beta]}\frac{d\underline{r}}{\underline{r}} - \frac{[1 + \lambda^2(\alpha + \beta)]}{2(1 + \lambda^2)}\,d\underline{S} = 0. \tag{2.21}$$

Along the negative characteristic lines,

$$d\underline{r} + [1 - (1 - 2\lambda^2)\alpha + \beta]\,d\underline{t} = 0, \tag{2.22}$$

the intermediate integral relation satisfied is, from (2.18),

$$d\beta - \frac{(\alpha - \beta)[1 + \lambda^2(\alpha + \beta)]}{[1 - (1 - 2\lambda^2)\alpha + \beta]}\frac{d\underline{r}}{\underline{r}} - \frac{[1 + \lambda^2(\alpha + \beta)]}{2(1 + \lambda^2)}\,d\underline{S} = 0; \tag{2.23}$$

and along the world-lines, or particle paths in the (r, t)-plane,

$$d\underline{r} = (1 - \lambda^2)(\alpha - \beta)\,d\underline{t}, \tag{2.24}$$

the relation satisfied is, from (2.19),

$$\underline{S} = \text{constant}. \tag{2.25}$$

## 3. The decaying spherical shock

When the leading blast shock from a spherical explosion has reached distances large compared with the size of the explosive charge, or when the peak overpressure immediately behind it is small compared with the undisturbed atmospheric pressure, it is pertinent to ask whether the actual conditions under which the energy was released by the 'detonation' continue to have any influence on the way in which the shock decays, or whether, in fact, the process of decay tends to depend less and less on the exact nature of the energy release. The answer to this question entails the study of an incompletely formulated problem in which the governing equations are those of section 2 above, and the relevant boundary conditions are those at the leading blast shock, and the end-condition that the shock ultimately decays to zero strength as its radius increases indefinitely. The problem is to draw conclusions from this data only, without recourse to the initial conditions of the detonation, or any other conditions, at any time, at boundaries nearer to the centre of the explosion than the leading shock.

Let the position of the leading blast shock at any time be denoted by the equivalent relations $R = R(t)$ or $T = T(r)$. In the notation of section 2 above, the time-development of the leading blast shock may be denoted by $\underline{T} = \underline{T}(\underline{R})$, and it is convenient, when the leading shock is weak, to write this relation in the form

$$\underline{T} = \underline{R} - e(\underline{R}) \tag{3.1}$$

where $e(\underline{R})$ is an unknown function of $\underline{R}$, to be determined so far as is possible.

Then from (3.1), if dashes denote differentiation with respect to $\underline{R}$,

$$\frac{d\underline{R}}{d\underline{T}} = \frac{1}{1 - e'} = M \tag{3.2}$$

is the Mach number of the shock in terms of the speed of sound in the undisturbed atmosphere, and thus, for a weak shock, $e'$ is small compared with unity. The shock conditions, giving the state of the air immediately behind the blast shock, are (see, for example, [4], pp. 74, 75),

$$\begin{aligned}
\underline{p} &= 1 + (1 + \lambda^2)(M^2 - 1) \\
\underline{u} &= \frac{(1 - \lambda^2)(M^2 - 1)}{M} \\
\frac{\rho}{\gamma} &= \frac{M^2}{1 + \lambda^2(M^2 - 1)} .
\end{aligned} \tag{3.3}$$

Substituting for $M$, from equation (3.2), the following results are obtained,

$$\begin{aligned}
\underline{p} &= 1 + 2(1 + \lambda^2)e' + 3(1 + \lambda^2)e'^2 + O(e'^3) \\
\underline{u} &= 2(1 - \lambda^2)e' + (1 - \lambda^2)e'^2 + O(e'^3) \\
\frac{\rho}{\gamma} &= 1 + 2(1 - \lambda^2)e' + (3 - 4\lambda^2)(1 - \lambda^2)e'^2 + O(e'^3) \\
\underline{a} &= 1 + 2\lambda^2 e' + \lambda^2 e'^2 + O(e'^3),
\end{aligned} \tag{3.4}$$

whence, from (2.15), (2.16) and (2.4),

$$\begin{aligned}
\alpha &= \frac{\underline{a} - 1}{2\lambda^2} + \frac{\underline{u}}{2(1 - \lambda^2)} = 2e' + e'^2 + O(e'^3) \\
\beta &= \frac{(\underline{a} - 1)}{2\lambda^2} - \frac{\underline{u}}{2(1 - \lambda^2)} = O(e'^3) \\
\underline{S} &= O(e'^3).
\end{aligned} \tag{3.5}$$

These are the required form of the leading shock boundary conditions in the notation adopted here.

The end-condition, that the shock tends to zero strength as the shock-radius increases indefinitely, is given by,

$$\underline{R} \to \infty; \qquad \epsilon' \to 0. \tag{3.6}$$

The problem then is to find, so far as is possible, the solutions of the governing equations, expressed characteristically in (2.20) to (2.25), satisfying the incomplete set of boundary conditions (3.5) and (3.6). Such solutions will apply to the air-flow immediately behind the leading blast shock, so far as no further shocks occur in the flow.

The method of solution adopted here may be described as one of 'analytic iteration'. It was found ultimately to have been proposed and used previously to some extent by H.A. Bethe [1]; Whitham [2] has adopted a different method of solution. Both methods are, in fact, devices for improving on the gross imperfections of 'acoustic' solutions to the problem, which fail on account of the divergence of the characteristics at infinity in the proper solution.

It is proposed to seek a solution only so far as second order terms in $\epsilon'$ at most. Since $\underline{S}$, at the leading shock is, by (3.5), only of the order of $\epsilon'^3$ and is constant along world-lines whose slope is of order $\epsilon'$, it follows that $\underline{S}$ is of the order $\epsilon'^3$ at all points of interest and may be neglected so far as a solution to second order terms in $\epsilon'$ is concerned. Since, moreover, by (3.5), $\beta$ is only of order $\epsilon'^3$ at the leading shock, whilst $\alpha$ is of order $\epsilon'$, a first approximation to equation (2.21) is

$$d(\alpha\,\underline{r}) = 0.$$

This is the approximation introduced by Bethe [1], and gives

$$\alpha\underline{r} = \text{constant} = 2\,\epsilon'_0\,\underline{R}_0$$

$$\text{or} \qquad \alpha = 2\,\epsilon'_0\,\underline{R}_0/\underline{r} \tag{3.7}$$

on the positive characteristic which overtakes the limiting shock at C (Figure 1). Using this result, a first approximation to the equation of the positive characteristic (2.20) is

$$d\underline{r}\left\{1 - \frac{2\,\epsilon'_0\,\underline{R}_0}{\underline{r}}\right\} - d\underline{t} = 0$$

giving, by integration,

$$\underline{r} - \underline{t} - 2\,\epsilon'_0\,\underline{R}_0 \log \underline{r} = \epsilon_0 - 2\,\epsilon'_0\,\underline{R}_0 \log \underline{R}_0 \tag{3.8}$$

for the positive characteristic which overtakes the leading shock at C.
[N.B. All logarithms are natural logarithms to base e.]

The negative characteristic (2.22) which intersects the leading shock at A (Figure 1) is, to a first approximation, simply

$$\underline{r} + \underline{t} = \underline{R}_A + \underline{T}_A = 2\,\underline{R}_A - \epsilon_A \tag{3.9}$$

If the characteristic which meets the leading shock at C cuts this negative characteristic at B, then, at B, to the same order of accuracy,

$$2\,\underline{r} - 2\,\underline{R}_A + \epsilon_A = 2\,\epsilon'_0\,\underline{R}_0 \log \underline{r} + \epsilon_0 - 2\,\epsilon'_0\,\underline{R}_0 \log \underline{R}_0 \tag{3.10}$$

by (3.8).

Along this negative characteristic, therefore,

$$2\frac{dr}{dR_o} = 2\log R_A(\epsilon_o' R_o)' - \frac{(\epsilon_o'^2 R_o^2 \log R_o)'}{\epsilon_o' R_o} + \text{higher order terms} \qquad (3.11)$$

by differentiation with respect to $R_o$, keeping $R_A$ constant.

A first approximation to the intermediate integral relation (2.23) is

$$d\beta = (\alpha - \beta)dr/r$$

$$\text{or} \qquad d(\beta r) = \alpha dr$$

whence, by integration, and using the boundary condition (3.5) on $\beta$ at the leading shock,

$$\beta r = \int_{R_A}^{r} \alpha dr$$

$$= \int_{R_A}^{r} \frac{2\epsilon_o' R_o}{r} dr \qquad \text{from (3.7)}$$

$$= \int_{R_A}^{R_o} \frac{2\epsilon_o' R_o}{r}\left\{\frac{dr}{dR_o}\right\} dR_o$$

$$= \int_{R_A}^{R_o} \frac{\epsilon_o' R_o}{R_A}\left[2\log R_A(\epsilon_o' R_o)' - \frac{(\epsilon_o'^2 R_o^2 \log R_o)'}{\epsilon_o' R_o}\right] dR_o$$

by (3.11)

$$= -\frac{(\epsilon_o' R_o)^2}{R_A}(\log R_o - \log R_A). \qquad (3.12)$$

Thus,

$$\beta = -\frac{(\epsilon_o' R_o)^2 (\log R_o - \log r)}{r^2} + \text{higher order terms} \qquad (3.13)$$

With these results the cycle may be repeated to a higher order of accuracy. Thus a better approximation to the relation (2.21) is

$$d\alpha + [\alpha - \beta - (1 - \lambda^2)\alpha^2]\frac{d\underline{r}}{\underline{r}} = 0$$

or

$$d(\alpha\underline{r}) = [\beta + (1 - \lambda^2)\alpha^2]\, d\underline{r},$$

giving

$$\alpha\underline{r} - 2\, e_0'\,\underline{R}_0 - e_0'^2\,\underline{R}_0 = \int_{\underline{R}_0}^{\underline{r}} \left[\frac{(e_0'\,\underline{R}_0)^2(\log \underline{r} - \log \underline{R}_0)}{\underline{r}^2} + \frac{4(1 - \lambda^2)e_0'^2\,\underline{R}_0^2}{\underline{r}^2}\right] d\underline{r}.$$

Hence

$$\alpha\underline{r} - 2\, e_0'\,\underline{R}_0 - e_0'^2\,\underline{R}_0 = (e_0'\,\underline{R}_0)^2\left[\frac{\log \underline{R}_0}{\underline{r}} - \frac{\log \underline{r}}{\underline{r}} - \frac{(5 - 4\lambda^2)}{\underline{r}}\right]_{\underline{R}_0}^{\underline{r}}$$

giving

$$\alpha = \frac{2\, e_0'\,\underline{R}_0}{\underline{r}} + \frac{(e_0'\,\underline{R}_0)^2}{\underline{r}^2}[\log \underline{R}_0 - \log \underline{r}] + \frac{2(3 - 2\lambda^2)e_0'^2\,\underline{R}_0}{\underline{r}} - (5 - 4\lambda^2)\frac{(e_0'\,\underline{R}_0)^2}{\underline{r}^2}$$

$$+ \text{ higher order terms.} \qquad (3.14)$$

A better approximation to the equation of the positive characteristic which meets the leading shock at O is

$$d\underline{r}[1 - \alpha + (1 - 2\lambda^2)\beta + \alpha^2] - d\underline{t} = 0,$$

giving

$$d\underline{r}\left[1 - \frac{2\, e_0'\,\underline{R}_0}{\underline{r}} - 2(1 - \lambda^2)(e_0'\,\underline{R}_0)^2\,\frac{(\log \underline{R}_0 - \log \underline{r})}{\underline{r}^2} - \frac{2(3 - 2\lambda^2)e_0'^2\,\underline{R}_0}{\underline{r}} + \frac{(9 - 4\lambda^2)(e_0'\,\underline{R}_0)^2}{\underline{r}^2}\right] - d\underline{t} = 0,$$

which integrates to give

$$\underline{r} - \underline{t} - 2\, e_0'\,\underline{R}_0 \log \underline{r} + 2(1 - \lambda^2)(e_0'\,\underline{R}_0)^2\left[\frac{\log \underline{R}_0 - \log \underline{r}}{\underline{r}} - \frac{1}{\underline{r}}\right]$$

$$- 2(3 - 2\lambda^2)e_0'^2\,\underline{R}_0 \log \underline{r} - (9 - 4\lambda^2)\frac{(e_0'\,\underline{R}_0)^2}{\underline{r}}$$

$$= e_0 - 2\, e_0'\,\underline{R}_0 \log \underline{R}_0 - 2(3 - 2\lambda^2)e_0'^2\,\underline{R}_0 \log \underline{R}_0 - (11 - 6\lambda^2)\, e_0'^2\,\underline{R}_0.$$

(3.15)

Since $\beta$, as given by (3.13) is only of the same order as the lower order terms of $\alpha$ given by (3.14), it is not necessary to improve the value of $\beta$ in the same way. A better approximation to the equation (3.9) for the negative characteristic which intersects the leading shock at A is, from (2.22),

$$d\underline{r}\,[1 + (1 - 2\lambda^2)\alpha] + d\underline{t} = 0.$$

Using the results (3.14) and (3.11), this leads on integration to the equation

$$\underline{r} + \underline{t} - \frac{(1 - 2\lambda^2)(\epsilon_o' \underline{R}_o)^2 (\log \underline{R}_o - \log \underline{r})}{\underline{r}} = 2\,\underline{R}_A - \epsilon_A. \qquad (3.16)$$

The required solutions are, therefore, given by the four relations (3.13) to (3.16), but it still remains to examine what restrictions, if any, are imposed on the, as yet, unknown function $\epsilon(\underline{R})$, with the help of the end-condition, $\epsilon'(\underline{R}) \to 0$ as $\underline{R} \to \infty$, which has not yet been used.

## 4. Excess energy and impulse per unit area

From the solution for the decaying spherical shock given in the last section, it is possible to calculate the impulse per unit area of the flow behind it, at least as far behind it as the flow is free from further shocks. Thus (see Figure 1), the impulse per unit area at a distance $R_A$ from the centre of the explosion, from the time $T_A$ at which the shock reaches this distance up to the time $t_D$ when the positive characteristic through C reaches this distance, is given by,

$$I = \int_{T_A}^{t_D} (p - p_o)\, dt$$

$$\text{or} \qquad \frac{I a_o}{p_o L} = \underline{I} = \int_{\underline{T}_A}^{\underline{t}_D} (\underline{p} - 1)\, d\underline{t}\,. \qquad (4.1)$$

Now since D is on the positive characteristic which meets the leading shock at C, by (3.15) or (3.8)

$$\underline{R}_A - \underline{t}_D - 2\,\epsilon_o' \underline{R}_o \log \underline{R}_A = \epsilon_o - 2\,\epsilon_o' \underline{R}_o \log \underline{R}_o$$
$$+ \text{ higher order terms} \qquad (4.2)$$

and, therefore, differentiating with respect to $\underline{R}_o$, keeping $\underline{R}_A$ constant,

$$\frac{d\underline{t}_D}{d\underline{R}_o} = \frac{(\epsilon_o'^2 \underline{R}_o^2 \log \underline{R}_o)'}{\epsilon_o' \underline{R}_o} - 2(\epsilon_o' \underline{R}_o)' \log \underline{R}_A + \text{higher order terms.} \qquad (4.3)$$

The relation (4.1) may now be integrated as follows:

$$\underline{I} = \int_{\underline{T}_A}^{\underline{t}_D} \left\{ \underline{a}^{2\gamma/(\gamma-1)} - 1 \right\} d\underline{t}, \text{by } (2.8)$$

$$= \int_{\underline{T}_A}^{\underline{t}_D} \left\{ [1 + \lambda^2(\alpha + \beta)]^{(1 + \lambda^2)/\lambda^2} - 1 \right\} d\underline{t}, \quad \text{by (2.14).}$$

$$\underline{I} = \int_{\underline{T}_A}^{\underline{t}_D} \left[ (1 + \lambda^2) \frac{2\, \epsilon_0' \underline{R}_0}{\underline{R}_A} + \cdots \right] d\underline{t}, \quad \text{by (3.13) and (3.14)}$$

$$= \int_{\underline{R}_A}^{\underline{R}_0} \left[ (1 + \lambda^2) \frac{2\, \epsilon_0' \underline{R}_0}{\underline{R}_A} + \cdots \right] \left[ \frac{(\epsilon_0'^2 \underline{R}_0^2 \log \underline{R}_0)'}{\epsilon_0' \underline{R}_0} - 2(\epsilon_0' \underline{R}_0)' \log \underline{R}_A \right] d\underline{R}_0,$$

by (4.3).

Thus

$$\underline{I} = \frac{2(1 + \lambda^2)}{\underline{R}_A} \epsilon_0'^2 \underline{R}_0^2 (\log \underline{R}_0 - \log \underline{R}_A) + \text{higher order terms.} \qquad (4.4)$$

This is a result of great importance, for it allows, at last, some consideration to be given to the possible forms of the unknown function $\epsilon(\underline{R})$, by examining what happens to the impulse per unit area $\underline{I}$, given by (4.4), as the point C on the leading shock tends to infinity.

First, in order to proceed at all, it is necessary to neglect the possibility of having an infinite number of shocks, behind the leading shock, which overtake it. In the absence of such an infinite array of secondary shocks, it follows that, for all values of $\underline{R}$ greater than some $\underline{R}_A$, there is a finite region of shock-free air-flow behind the leading shock. It may be remarked here, in support of this assumption, that observations do not show any secondary shocks at all which overtake the leading shock.

There are then three possibilities for the form of the unknown function $\epsilon(\underline{R})$.

First, if $\epsilon(\underline{R})$ were a function such that

$$\text{as} \quad \underline{R} \to \infty, \qquad \epsilon'^2 \underline{R}^2 \log \underline{R} \to \infty$$

then, by (4.4), the impulse per unit area in the air-flow behind the leading shock would also tend to infinity at some finite subsequent time, at all large distances $\underline{R}_A$. For (see Figure 1) any further shock which intervened at a time $\underline{t}_D$, corresponding to a positive characteristic meeting the shock at a finite point C, must necessarily overtake the leading shock earlier than the point C, and this is a possibility which has already been neglected. The possibility that $\epsilon'^2 \underline{R}^2 \log \underline{R} \to \infty$, which leads to infinite positive impulses per unit area at finite distances, is also neglected.

Second, if $\epsilon(\underline{R})$ were a function such that

$$\text{as} \quad \underline{R} \to \infty, \qquad \epsilon'^2 \underline{R}^2 \log \underline{R} \to 0,$$

then the function

$$\epsilon_0'^2 \underline{R}_0^2(\log \underline{R}_0 - \log \underline{R}_A)$$

would be zero for $\underline{R}_0 = \underline{R}_A$

positive for $\underline{R}_0 > \underline{R}_A$

and would $\to 0$ as $\underline{R}_0 \to \infty$.

It would therefore have a maximum at some finite value $\underline{R}_{0_M}$. However, by (4.3), at the point $D_M$ corresponding to $O_M$,

$$\frac{dt_D}{d\underline{R}_0} = \frac{\left[\epsilon_0'^2 \underline{R}_0^2(\log \underline{R}_0 - \log \underline{R}_A)\right]'}{\epsilon_0' \; \underline{R}_0} = 0$$

to the order of equation (4.3), since $\epsilon_0'^2 \underline{R}_0^2(\log \underline{R}_0 - \log \underline{R}_A)$ has a maximum value at $D_M$, and beyond $D_M$, $dt_D/d\underline{R}_0$ would become negative. This possibility is therefore also neglected.

There remains, then, only the third possibility that $\epsilon(\underline{R})$ is a function such that,

$$\text{as } \underline{R} \to \infty, \quad \epsilon'^2 \underline{R}^2 \log \underline{R} \to \text{a finite non-zero quantity, say } k^2/4. \tag{4.5}$$

Then, as $\underline{R} \to \infty$,

$$\epsilon' \to \frac{k}{2\,\underline{R}\,\log^{\frac{1}{2}} \underline{R}}, \tag{4.6}$$

$$\epsilon \to k \log^{\frac{1}{2}} \underline{R} + K, \tag{4.7}$$

and it is seen that this possibility is consistent with the end-condition that $\epsilon' \to 0$ as $\underline{R} \to \infty$. With this possibility, the expression (4.4) for the impulse per unit area becomes, as $\underline{R}_0 \to \infty$

$$\underline{I} \to \frac{(1+\lambda^2)k^2}{2\underline{R}_A} + \text{higher order terms} \tag{4.8}$$

and thus as $\underline{R}_A \to \infty$,

$$\underline{I}\,\underline{R}_A \to (1+\lambda^2)k^2/2\,. \tag{4.9}$$

Turning now to considerations of the energy in the air-flow behind the leading shock, the energy transported across the surface of a given sphere $r = R_A$ from the time $T_A$ (Figure 1) at which the shock reaches this sphere up to the time $t_D$ when the positive characteristic through C reaches this sphere, consists of the energy of the mass transported across this sphere, together with the work done at the surface of the sphere by fluid inside the sphere <u>at any time</u> on fluid outside the sphere <u>at any time</u>. It is thus given by

$$E_1 = \int_{T_A}^{t_D} 4\pi R_A^2 \left\{\left(\frac{\rho u^2}{2} + \frac{p}{\gamma - 1}\right) + p\right\} u \, dt$$

or

$$\frac{E_1}{p_o L^3} = \underline{E}_1 = \frac{2\pi \underline{R}_A^2(1 + \lambda^2)}{\lambda^2} \int_{\underline{T}_A}^{\underline{t}_D} \left[\underline{p} + \lambda^2 \underline{\rho}\, \underline{u}^2/(1 + \lambda^2)\right] \underline{u} \, d\underline{t}. \quad (4.10)$$

The integration may be performed by methods similar to those used for integrating the expression for $\underline{I}$, and, without repeating the details, the result is

$$\underline{E}_1 = \frac{4\pi(1 - \lambda^4)}{\lambda^2} \underline{R}_A \, c_o'^2 \, \underline{R}_o^2(\log \underline{R}_o - \log \underline{R}_A)$$
$$+ \text{ higher order terms.} \quad (4.11)$$

Thus, by (4.5), as $\underline{R}_o \to \infty$,

$$\underline{E}_1 \to \frac{k^2\pi(1 - \lambda^4)}{\lambda^2} \underline{R}_A \quad (4.12)$$

and hence the surprising result that,

$$\text{as} \quad \underline{R}_A \to \infty, \qquad \underline{E}_1 \to \infty,$$

in such a way that

$$\underline{E}_1/\underline{R}_A \to k^2\pi(1 - \lambda^4)/\lambda^2. \quad (4.13)$$

Alternatively the energy $E_1$ should also be equal to the excess energy at the time $t_D$, over and above the atmospheric energy present before the arrival of the blast, contained in the spherical annulus between the leading shock $r = R_A'$, and the sphere $r = r_D$ (see Figure 1). This excess energy over atmospheric is given by

$$E_1 = \int_{r_D}^{R_A'} 4\pi r^2 \rho \left[\frac{u^2}{2} + \frac{a^2}{\gamma(\gamma - 1)}\right] dr - \frac{4\pi}{3}\left\{R_A'^3 - r_D^3\right\} \frac{p_o}{(\gamma - 1)}$$

or

$$\frac{E_1}{p_o L^3} = \underline{E}_1 = \frac{2\pi(1 - \lambda^2)}{\lambda^2} \int_{\underline{r}_D}^{\underline{R}_A'} \underline{r}^2 \left[\underline{p} - 1 + \lambda^2 \underline{\rho}\, \underline{u}^2/(1 - \lambda^2)\right] d\underline{r}. \quad (4.14)$$

Again, the integration may be performed by similar methods to those used previously, after deriving an expression for $d\underline{y}/d\underline{R}_0$ along A'D (Figure 1), and the result is

$$\underline{E}_1 = \frac{4\pi(1 - \lambda^4)}{\lambda^2} \underline{R}_A\, c_0'^2\, \underline{R}_0^2(\log \underline{R}_0 - \log \underline{R}_A)$$

$$+ \text{ higher order terms} \qquad (4.15)$$

in agreement with (4.11).

The arguments applied to the positive impulse per unit area $\underline{I}$, (4.4), in order to determine the most probable form of the function $\epsilon(\underline{R})$, could, of course, have been applied equally well to the energy $\underline{E}_1$.

## 5. The limiting positive characteristic terminating the shock-decay phase of the air-flow behind a blast shock

In this section, some further consequences of the restrictions, (4.5), (4.6), (4.7), placed on the function $\epsilon(\underline{R})$ in the previous section, are examined.

Under these restrictions, the equation (3.15) or (3.8) of the positive characteristic which overtakes the leading shock at C reduces to

$$\underline{r} - \underline{t} - k \log \underline{y}/\log^{\frac{1}{2}} \underline{R}_0 + \text{higher order terms} = K + 0\left(\frac{1}{\underline{R}_0}\right). \qquad (5.1)$$

In the limit as $\underline{R}_0 \to \infty$ then, this gives (cf. 3.15), for the limiting positive characteristic which just fails to overtake the leading shock in finite distance, the equation

$$\underline{r} - \underline{t} + k^2(1 - \lambda^2)/2\underline{r} = K. \qquad (5.2)$$

This may be compared with the result obtained by Whitham [3] for the limiting characteristic, namely, in the present notation,

$$\underline{r} - \underline{t} + 0\ (\log \underline{y}/\underline{r}) = K. \qquad (5.2a)$$

The method used here appears to show that the term of order $\log \underline{y}/\underline{r}$ envisaged by Whitham has, in fact, zero coefficient, and that the next surviving term is only of order $1/\underline{r}$, and is determinable by the method used here.

This limiting characteristic (VW in Figure 1) divides the whole fluid flow behind the leading shock into two parts, of important significance. For the flow behind the limiting characteristic VW cannot exert any influence at all on the shock decay process, otherwise than through the medium of a shock overtaking VW. But any shock which overtakes VW must overtake the leading shock in finite distance, since its slope must be greater at any point, than the slope of the positive characteristic there. The possibility of an infinite number of such shocks has been ruled out, and attention is being confined to distances greater than that at which the last of such secondary shocks, if any, overtakes the leading shock.

Hence no flow behind the limiting characteristic VW can influence the shock decay process, and the shock decay process is therefore controlled entirely by the flow between the leading shock AC and the limiting positive characteristic VW. This part of the flow will be called <u>the shock-decay phase of the blast wave.</u>

The equation (3.1) of the leading shock is, by (4.7),

$$\underline{R} = \underline{T} + k \log^{\frac{1}{2}} \underline{R} + K + \text{higher order terms}. \qquad (5.3)$$

The peak overpressure immediately behind the leading shock is, by (4.6) and (3.4),

$$\underline{p} - 1 \;=\; \frac{k(1 + \lambda^2)}{\underline{R}\, \log^{\frac{1}{2}} \underline{R}} + \text{higher order terms.} \tag{5.4}$$

The duration of the shock-decay phase at $r = R_A$ (AZ in Figure 1) is given by

$$\begin{aligned} \underline{\tau}_A \;&=\; \underline{T}_Z - \underline{T}_A \\ &=\; \left[\underline{R}_A - K + k^2(1 - \lambda^2)/2\underline{R}_A\right] - \underline{T}_A, \end{aligned}$$

since Z is on the limiting characteristic (5.2).

Thus, by (5.3),

$$\underline{\tau}_A \;=\; k \log^{\frac{1}{2}} \underline{R}_A + \text{higher order terms.} \tag{5.5}$$

Behind the leading blast shock, the particle velocity $\underline{u}$ is, by (2.13), (3.13), (3.14) and (4.6),

$$\underline{u} \;=\; (1 - \lambda^2)k/\underline{r}\, \log^{\frac{1}{2}} \underline{R}_0 + \text{higher order terms} \tag{5.6}$$

and the overpressure $(\underline{p} - 1)$ is, by (2.8), (2.14), (3.13), (3.14) and (4.6),

$$\underline{p} - 1 \;=\; k(1 + \lambda^2)/\underline{r}\, \log^{\frac{1}{2}} \underline{R}_0 + \text{higher order terms.} \tag{5.7}$$

At the limiting characteristic, (3.14) gives, as $\underline{R}_0 \to \infty$,

$$\alpha = \frac{k^2}{4\underline{r}^2} + \text{possible higher order terms,} \tag{5.8}$$

and (3.13) gives, as $\underline{R}_0 \to \infty$,

$$\beta = -\frac{k^2}{4\underline{r}^2} + \text{possible higher order terms.} \tag{5.9}$$

Thus, at the limiting characteristic, the overpressure,

$$\underline{p} - 1 \;=\; (1 + \lambda^2)(\alpha + \beta) + \text{higher order terms,}$$

is of lower order than $1/\underline{r}^2$, and the particle velocity is

$$\underline{u} = (1 - \lambda^2)(\alpha - \beta) = \frac{k^2(1 - \lambda^2)}{2\underline{r}^2} + \text{possible higher order terms.} \tag{5.10}$$

Since, at the limiting characteristic, the overpressure is not much different from zero, the shock-decay phase as defined here is relatively not much different from the positive overpressure phase of the flow. However, if, in fact, the overpressure at the rear of the shock-decay phase is still greater than zero, it is possible for a secondary shock, behind the limiting characteristic, and never overtaking it, to enter the positive overpressure phase, increase the overpressure and thus prolong the positive overpressure phase. In explosions from conventional explosives, such a secondary shock is observed [5, 6] and its inception and development have been investigated [7, 8].

To complete the results for the shock-decay phase of the motion it may be repeated that the impulse per unit area in the shock-decay phase (4.8) is

$$\underline{I} = \frac{(1 + \lambda^2)k^2}{2\underline{R}} + \text{higher order terms;} \tag{5.11}$$

and the excess energy over atmospheric in the shock-decay phase (4.16) is

$$E_1 = \frac{k^2 \pi (1 - \lambda^4)}{\lambda^2} \underline{R} + \text{higher order terms.} \tag{5.12}$$

## 6. A solution to higher order of accuracy

The considerations of section 4 led to the conclusion that the only restriction on the function $\epsilon(\underline{R})$ for a spherically decaying shock is

$$\underline{R} \to \infty; \qquad \epsilon' \to \frac{k}{2\,\underline{R}\,\log^{\frac{1}{2}}\underline{R}}$$

$$\text{or} \qquad \epsilon \to k\log^{\frac{1}{2}}\underline{R} + K.$$

There seems therefore no possibility of determining the function $\epsilon$ more precisely without appeal to the initial conditions of the explosion, or to boundary conditions within the leading spherical shock, or without making some hypothesis.

The boundary conditions at the leading shock (3.5) are expressible as power series in $\epsilon'$ when $\epsilon'$ is small. On the other hand, the fundamental length L is so far arbitrary, and if, for instance, a different length $L_1$ were used, and $\underline{R}_1$ is written for $R/L_1$,

$$\log^{\frac{1}{2}}\underline{R}_1 = \log^{\frac{1}{2}}\left\{\frac{R}{L_1}\right\} = \left[\log\underline{R} + \log\left\{\frac{L}{L_1}\right\}\right]^{\frac{1}{2}}$$

$$= \log^{\frac{1}{2}}\underline{R}\left[1 + \frac{\log(L/L_1)}{2\log\underline{R}} - \frac{\log^2(L/L_1)}{8\log^2\underline{R}} + \ldots\ldots\right] \tag{6.1}$$

so that an expansion in powers of $1/\log\underline{R}$ would be introduced into the expressions for $\epsilon$ and $\epsilon'$, by a change in the fundamental length L.

On this basis, the hypothesis is made that, for spherical explosions, $\epsilon$ may be represented by a double series of the form

$$\begin{aligned}\epsilon = {} & k\log^{\frac{1}{2}}\underline{R} - m\log^{-\frac{1}{2}}\underline{R} - l\log^{-3/2}\underline{R} + \ldots\ldots\ldots \\ & + \frac{1}{\underline{R}}f_1(\log^{\frac{1}{2}}\underline{R}) \\ & + \frac{1}{\underline{R}^2}f_2(\log^{\frac{1}{2}}\underline{R}) \\ & + \ldots\ldots\ldots\end{aligned} \tag{6.2}$$

in which the origin of time has been chosen so that $K = 0$.

It follows from this hypothesis that, at large distances $\underline{R}$, an attempt to derive a higher order approximation than from the first term only of $\epsilon$, by including a finite number, say two, of higher order terms, must be directed not towards including terms of the order $\epsilon'$ and $\epsilon'^2$ times the leading term but rather towards an approximation of the form

$$\epsilon = k \log^{\frac{1}{2}} \underline{R} - m \log^{-\frac{1}{2}} \underline{R} - l \log^{-3/2} \underline{R}. \qquad (6.3)$$

Now L, the fundamental length, is arbitrary, and it follows from (6.1) that L may be chosen definitely so that the coefficient m = 0. With this definite value of L then, the approximation becomes,

$$\epsilon = k \log^{\frac{1}{2}} \underline{R} \left\{1 - \frac{l}{k \log^2 \underline{R}}\right\} \qquad (6.4)$$

and all further results given are based on the above hypothesis, and with this definite choice of L. The expression (6.4) contains, then, three constants, L, l, and k which, if the hypothesis is justified, should be determinable by comparison with experimental observations.

With the form (6.4), and neglecting terms of higher order than $1/\log^2 \underline{R}$ times the leading term as well as all terms of order $\epsilon'$ or higher, compared with the leading term, the results for the shock-decay phase of the blast motion at a distance $\underline{R}$ from the centre of explosion are:

$$\epsilon' = \frac{k}{2 \underline{R} \log^{\frac{1}{2}} \underline{R}} \left\{1 + \frac{3l}{k \log^2 \underline{R}}\right\}; \qquad (6.5)$$

Shock peak overpressure (5.4),

$$\underline{P} - 1 = \frac{k(1+\lambda^2)}{\underline{R} \log^{\frac{1}{2}} \underline{R}} \left\{1 + \frac{3l}{k \log^2 \underline{R}}\right\}; \qquad (6.6)$$

General overpressure in the shock-decay phase (5.7),

$$\underline{p} - 1 = \frac{k(1+\lambda^2)}{\underline{R} \log^{\frac{1}{2}} \underline{R}_o} \left\{1 + \frac{3l}{k \log^2 \underline{R}_o}\right\}; \qquad (6.7)$$

Duration of shock-decay phase (5.5),

$$\underline{\tau} = k \log^{\frac{1}{2}} \underline{R} \left\{1 - \frac{l}{k \log^2 \underline{R}}\right\}; \qquad (6.8)$$

Shock-decay impulse per unit area (5.11),

$$\underline{I} = k^2(1+\lambda^2)/2\underline{R} \text{ with no term in } 1/\underline{R} \log^2 \underline{R}; \qquad (6.9)$$

Equation of positive characteristic which overtakes leading shock at O, (3.15);

$$\underline{r} - \underline{t} - \frac{k \log \underline{r}}{\log^{\frac{1}{2}} \underline{R}_o} \left\{1 + \frac{3l}{k \log^2 \underline{R}_o}\right\} = - \frac{4l}{\log^{3/2} \underline{R}_o}; \qquad (6.10)$$

Relation between peak overpressure, shock-decay impulse per unit area, and duration of shock-decay phase, (6.6), (6.8) and (6.9),

$$(\underline{P} - 1)\underline{\tau} = 2\underline{I} \left\{1 + \frac{2l}{k \log^2 \underline{R}}\right\}. \qquad (6.11)$$

## 7. The form of the pressure-pulse in a spherical blast wave

The results given in the previous section 6 enable an expression to be obtained, for the first time so far as the author is aware, for the ultimate pressure-time relation behind a spherical blast wave, at sufficiently large distances from the centre of the explosion.

For, from (6.6), the peak overpressure at the shock, when it is at distance $R_A$, is

$$P - 1 = \frac{k(1 + \lambda^2)}{R_A \log^{\frac{1}{2}} R_A}\left\{1 + \frac{31}{k \log^2 R_A}\right\} \tag{7.1}$$

and subsequently, behind the shock, at the same distance $R_A$ from the centre of the explosion, the overpressure is generally, by (6.7),

$$p - 1 = \frac{k(1 + \lambda^2)}{R_A \log^{\frac{1}{2}} R_o}\left\{1 + \frac{31}{k \log^2 R_o}\right\} \tag{7.2}$$

at a time $t_D$ (see Figure 1) at which the positive characteristic overtaking the leading shock at C reaches the distance $R_A$.

Since D is on the positive characteristic which overtakes the leading shock at C, by (6.10),

$$R_A - t_D = \frac{k \log R_A}{\log^{\frac{1}{2}} R_o}\left\{1 + \frac{31}{k \log^2 R_o}\right\} - \frac{41}{\log^{3/2} R_o}, \tag{7.3}$$

whilst if $\tau$ is the shock-decay duration AZ (Figure 1), by (6.8),

$$\tau = k \log^{\frac{1}{2}} R_A \left\{1 - \frac{1}{k \log^2 R_A}\right\}, \tag{7.4}$$

and the relation between $R_A$ and $T_A$ at the leading shock is, by (3.1) and (6.4),

$$R_A - T_A = k \log^{\frac{1}{2}} R_A \left\{1 - \frac{1}{k \log^2 R_A}\right\}. \tag{7.5}$$

From (7.3) and (7.5), by subtraction,

$$t_D - T_A = k \log^{\frac{1}{2}} R_A \left\{1 - \frac{\log^{\frac{1}{2}} R_A}{\log^{\frac{1}{2}} R_o}\right\} - \frac{1}{\log^{3/2} R_A}\left\{1 + \frac{3 \log^{3/2} R_A}{\log^{3/2} R_o} - \frac{4 \log^{3/2} R_A}{\log^{3/2} R_o}\right\}. \tag{7.6}$$

Thus, from (7.4),

$$\left\{1 - \frac{t_D - T_A}{\tau}\right\} = \frac{\log^{\frac{1}{2}} R_A}{\log^{\frac{1}{2}} R_o}\left\{1 + \frac{31}{k \log^2 R_o} - \frac{41}{k \log R_A \log R_o} + \frac{1}{k \log^2 R_A}\right\}, \tag{7.7}$$

whereas, from (7.2) and (7.1),

$$\frac{p - 1}{P - 1} = \frac{\log^{\frac{1}{2}} R_A}{\log^{\frac{1}{2}} R_o}\left\{1 + \frac{31}{k \log^2 R_o} - \frac{31}{k \log^2 R_A}\right\} \tag{7.8}$$

Thus, from (7.7) and (7.6),

$$\frac{\underline{p}-1}{\underline{P}-1} = \left\{1-\frac{(t_D-\underline{T}_A)}{\underline{\tau}}\right\}\left\{1-\frac{41}{k\log^2\underline{R}_A}+\frac{41}{k\log\underline{R}_A\log\underline{R}_o}\right\}$$

$$= \left\{1-\frac{(t_D-\underline{T}_A)}{\underline{\tau}}\right\}\left\{1-\frac{41}{k\log^2\underline{R}_A}\left[1-\left\{1-\frac{t_D-\underline{T}_A}{\underline{\tau}}\right\}^2\right]\right\} \qquad (7.9)$$

to the present order of approximation.

If, in the present context, time is measured from the instant the shock reaches the radius $R_A$, (7.9) may be written simply,

$$\frac{p-p_o}{P-p_o} = \left\{1-\frac{t}{\tau}\right\}\left[1-\frac{41}{k\log^2\underline{R}_A}\left\{\frac{2t}{\tau}-\frac{t^2}{\tau^2}\right\}\right]. \qquad (7.10)$$

This may be compared with the empirical form first suggested by Friedlander [9], namely,

$$\frac{p-p_o}{P-p_o} = \left\{1-\frac{t}{T}\right\}e^{-t/T}, \qquad (7.11)$$

or with variations on the Friedlander form [cf. 5, 6] such as

$$\frac{p-p_o}{P-p_o} = \left\{1-\frac{t}{T}\right\}e^{-c_1 t/T}, \qquad (7.12)$$

or

$$\frac{p-p_o}{P-p_o} = \left\{1-\frac{t}{T}\right\}^{c_2}e^{-c_2 t/T}. \qquad (7.13)$$

In the relations (7.11), (7.12) and (7.13), $c_1$ and $c_2$ are constants, and T is the duration of the positive overpressure phase.

To obviate the experimental difficulties associated with the measurement of quantities connected with the entire positive phase or the entire shock-decay phase as introduced in section 5, it is convenient to define a general time-interval $\tau_f$, after the arrival of the leading blast shock at a given radius $R_A$, as the elapsed time at which the overpressure has fallen to a fraction f of its peak value. The positive characteristic through the particular point $D_1$ (Figure 1) corresponding to $(\underline{R}_A, \underline{T}_A+\tau_f)$ overtakes the leading shock at the particular point $C_1$ satisfying, by (7.8),

$$f = \frac{\log^{\frac{1}{2}}\underline{R}_A}{\log^{\frac{1}{2}}\underline{R}_{C_1}}\left\{1+\frac{31}{k\log\underline{R}_{C_1}}-\frac{31}{k\log^2\underline{R}_A}\right\}, \qquad (7.14)$$

whilst, from (7.6),

$$\underline{\tau}_f = k\log^{\frac{1}{2}}\underline{R}_A\left\{1-\frac{\log^{\frac{1}{2}}\underline{R}_A}{\log^{\frac{1}{2}}\underline{R}_{C_1}}\right\}-\frac{1}{\log^{3/2}\underline{R}_A}\left\{1+\frac{3\log^{3/2}\underline{R}_A}{\log^{3/2}\underline{R}_{C_1}}-\frac{4\log^{3/2}\underline{R}_A}{\log^{3/2}\underline{R}_{C_1}}\right\}. \qquad (7.15)$$

From (7.14) and (7.15), to the present order of approximation,

$$\underline{\tau}_f = k(1-f)\log^{\frac{1}{2}}\underline{R}_A\left[1-\frac{1}{k\log^2\underline{R}_A}(1+2f)^2\right]. \qquad (7.16)$$

Of particular interest is the special case when the fraction f has the value $f_x = (\sqrt{3} - 1)/2$. In this particular case

$$(1 + 2f_x)^2 = 3 \tag{7.17}$$

and

$$\underline{\tau}_x = \frac{k(3 - \sqrt{3})}{2} \log^{\frac{1}{2}} \underline{R}_A \left\{1 - \frac{3l}{k \log^2 \underline{R}_A}\right\}, \tag{7.18}$$

so that, from (7.4),

$$\frac{\underline{\tau}_x}{\underline{\tau}} = \frac{(3 - \sqrt{3})}{2} \left\{1 - \frac{2l}{k \log^2 \underline{R}_A}\right\} \tag{7.19}$$

and (6.11) may be written

$$(\underline{P} - 1)\,\underline{\tau}_x = (3 - \sqrt{3})\,\underline{I}. \tag{7.20}$$

The relation (7.20) appears to be the most useful relation of its kind, for further analysis gives, in general,

$$\underline{I}_f = \frac{k^2(1 + \lambda^2)(1 - f^2)}{2\underline{R}_A}\left[1 - \frac{6lf^2}{k \log^2 \underline{R}_A}\right], \tag{7.21}$$

as a generalisation of (5.11), and thus

$$(\underline{P} - 1)\,\underline{\tau}_f = \frac{2\underline{I}_f}{(1 + f)}\left[1 + \frac{2\,l(1 - f)^2}{k \log^2 \underline{R}_A}\right], \tag{7.22}$$

as a generalisation of (6.11). It is clear from (7.22) that the special case $f = f_x$ has no particular merit so far as this relation is concerned, and that there is no other special value of f (apart from the trivial exception $f = 1$) which will reduce (7.22) to the form

$$\frac{(\underline{P} - 1)\,\underline{\tau}_f}{\underline{I}_f} = \text{constant.}$$

Finally, the generalisation of (5.12) is obtained as,

$$\underline{E}_{1f} = \frac{k^2\pi(1 - \lambda^4)}{\lambda^2}(1 - f^2)\,\underline{R}_A\left[1 - \frac{6lf^2}{k \log^2 \underline{R}_A}\right]. \tag{7.23}$$

## 8. Comparison with experimental observations

In order to facilitate comparison with experimental observations, it is desirable to derive some further results of the theory in terms of actual physical quantities rather than the reduced non-dimensional quantities which have been used in the development of the theory.

Thus, equation (6.9) may be written

$$k^2L^2 = \frac{2\,I\,Ra_0}{P_0(1 + \lambda^2)}, \tag{8.1}$$

and equation (7.20) may be written

$$k^2L^2 = \frac{(\underline{P} - 1)\,\tau_x\,Ra_0\,(\sqrt{3} + 1)}{(1 + \lambda^2)\sqrt{3}} \tag{8.2}$$

in which $(\underline{P} - 1) = (P - p_0)/p_0$ is the peak overpressure measured in atmospheres and is left in the reduced notation for simplicity.

From (6.6) and (6.8)

$$(\underline{P} - 1)\, \underline{\tau}^3\, \underline{R} = k^4(1 + \lambda^2)(\log \underline{R}),$$

and this may now be written

$$k^4L^4 \ (\log R - \log L) \ = \ \frac{(\underline{P} - 1)\ \tau^3\ R\ a_0^3}{(1 + \lambda^2)}. \tag{8.3}$$

Similarly, from (6.8) and (7.18)

$$\frac{\underline{\tau}^3}{\underline{\tau}_{\mathfrak{M}}} = \frac{(\sqrt{3} + 1)}{\sqrt{3}}\, k^2 \log \underline{R},$$

and this may now be written

$$k^2L^2 \ (\log R - \log L) \ = \ \frac{\tau^3\ a_0^2(3 - \sqrt{3})}{2\tau_{\mathfrak{M}}}. \tag{8.4}$$

Eliminating kL between (8.3) and (8.4) gives the result

$$\log L \ = \ \log R - \frac{3(2 - \sqrt{3})}{2}\ \frac{\tau^3\ a_0(1 + \lambda^2)}{R(\underline{P} - 1)\ \tau_{\mathfrak{M}}^2}. \tag{8.5}$$

Finally, when these five relations have been found to yield consistent values of k and L, by comparison with experimental observations, the following three relations may be used to determine l, and thus further assess the consistency of the agreement between theory and observations.

From (6.6)

$$l = \frac{(\underline{P} - 1)\ \underline{R}\ \log^{3/2} \underline{R}}{3(1 + \lambda^2)} - \frac{k \log^2 \underline{R}}{3}. \tag{8.6}$$

From (6.8)

$$l = k \log^2 \underline{R} - \underline{\tau} \log^{3/2} \underline{R}, \tag{8.7}$$

and from (7.18)

$$l = \frac{k \log^2 \underline{R}}{3} - \frac{(\sqrt{3} + 1)}{3\sqrt{3}}\ \tau_{\mathfrak{M}} \log^{3/2} \underline{R}. \tag{8.8}$$

Potter and Jarvis [5, 6] have made observations of explosions from bare spherical charges of TNT and RDX/TNT 60/40 which are sufficiently comprehensive for a comparison, on the above lines, with the theory developed here. Their results are given in terms of the common method of scaling, using a factor $W^{\frac{1}{3}}$ (W = charge mass in pounds). Here, W is put equal to one pound in their results, so that, in fact, their observations are scaled to one pound of explosive at sea-level, and the comparison is made on this basis. On this scale, their observations extend from about 4 ft. to 50 ft. from the centre of the explosion, and their peak overpressure measurements were fitted, over the complete range, by the method of least squares, to the form

$$\underline{P} - 1 = a/R + b/R^2 + c/R^3.$$

Potter and Jarvis have, of course, analysed their records in terms of the usual positive overpressure phase. In the case of RDX/TNT they observed a secondary shock which entered the positive phase at about 15 ft. from the centre of the explosion, and at greater distances their quoted values of positive duration include a small extrapolation to give an estimate of what the positive duration would have been in the absence of a second shock. Their values of positive impulse per unit area, however, include only the impulse in the primary wave. The comparison made here rests entirely on the assumption that these results of Potter and Jarvis may be taken as a good approximation to values for the shock-decay phase introduced in the present theory.

It may seem surprising that the results of Potter and Jarvis are sufficiently comprehensive to include measurements of $\tau_x$, (7.18). In fact, Potter and Jarvis have measured, on their records, the time $\tau_f$, (7.16), for the fraction $f = 1/e$. By a remarkable numerical coincidence,

$$\frac{1}{f_x} = \frac{2}{\sqrt{3} - 1} = \sqrt{3} + 1 = 2.73205,$$

as compared with $e = 2.71828$, or

$$(1 + \tfrac{2}{e})^2 = 3.0129,$$

as compared with $(1 + 2f_x)^2 = 3$.

The values of $\tau_{1/e}$, (which they name the 'decay-constant'), given by Potter and Jarvis, have therefore been used as values of $\tau_x$. The author, of course, disagrees violently with their assertion that 'the choice of the factor $1/e$ is quite arbitrary, and 0.5 or 0.25 would have been almost as good'!

Table (8.1) shows the results for one pound of RDX/TNT 60/40 at sea-level. Columns (6) and (7) show the two values for $k^2L^2$ given by the relations (8.1) and (8.2). Column (8) shows the value of L given by the relation (8.5). Figure 2 shows the plots of the relations (8.3) and (8.4), on semilogarithmic graph paper, and the straight lines corresponding to the values $k = 0.716$, $L = 3.8$ ft. are also drawn. Finally, using these values of k and L, the three values of l given by the relations (8.6), (8.7) and (8.8) are given in columns (9), (10) and (11) of Table 8.1, being denoted respectively by $l(P)$, $l(\tau)$ and $l(\tau_x)$.

Table (8.2) and Figure 3 give the corresponding comparison with the results for one pound of TNT at sea-level, and, for TNT, the values $k = 0.6025$, $L = 4.5$ feet have been chosen. At the greatest distances from the centre of the explosion, the observations naturally become less reliable, and the least squares fit of the mean curve by Potter and Jarvis, being chosen as an optimum over the whole range of observations, is probably not so good at the greatest distances, as a best fit over a shorter range of larger distances. It is not surprising, therefore, that the good agreement shown in Tables 8.1 and 8.2 and Figures 2 and 3 up to distances of about 35 feet, is not so well maintained at 40 and 50 feet. It appears that the fitted mean values of $(\underline{P} - 1)$ and I are the least reliable, and amended values of these quantities are shown in brackets in Tables 8.1 and 8.2 and Figures 2 and 3, to give some idea of the changes in these quantities which would give agreement with the theory given here. These changes are quite small.

Finally, as an alternative to (8.2), the relation (7.20) may be written in the form,

$$(P - p_0)\,\tau_x = (3 - \sqrt{3})\,I, \qquad (8.9)$$

and thus constitutes an overall check on the hypothesis of section 7, when tested against experimental observations.

The values of $(P - p_0)\tau_x/(3 - \sqrt{3})\,I$ are given in column 12 of Tables 8.1 and 8.2. It is seen that, in both cases, as R increases, this quantity tends rapidly and monotonically to unity.

## 9. Remarks

(a) It needs to be stressed that the hypothesis made in section 6 and the consequent results given in sections 7 and 8 are not necessarily sound, and other similar hypotheses need to be investigated before any firm conclusions can be drawn. From the results given in section 8, however, it is seen that this hypothesis appears to be in reasonable accord with experimental observations, particularly in the sense that the observations used are far more extensive than the minimum requirements for determining only three constants L, k, l, and yield consistent values whenever the constants are determined by alternative methods from different types of observation. On the other hand, the solution which is being compared with experiment is an asymptotic one, and the vital determination of the constants depends therefore on the measurements at large distances, where the observations are naturally at their weakest. For this reason the numerical constants chosen to fit the observations, in section 8, should not be regarded as determined to any high order of accuracy.

(b) It is of interest to compare the fit with observations of the higher order asymptotic solution proposed here with the corresponding fit of the 'zero order' solution involving only the leading terms. Kirkwood and Brinkley [2 et al ], who gave a solution to the whole development of a spherical explosion by making a physical assumption, obtained the asymptotic solution for the peak overpressure from the limiting form of their differential equations, as the leading term only of the solution given here, i.e. in the present notation

$$\underline{P} - 1 = \frac{k_1(1 + \lambda^2)}{\underline{R}_1 \log^{\frac{1}{2}} \underline{R}_1} \tag{9.1}$$

in terms of a fundamental length $L_1$, where $\underline{R}_1 = R/L_1$.

The corresponding solution for other quantities to the same order of accuracy, by the methods given here, would thus be

$$\underline{I} = \frac{k_1^2(1 + \lambda^2)}{2\underline{R}_1} \tag{9.2}$$

$$\underline{\tau} = k_1 \log^{\frac{1}{2}} \underline{R}_1 \tag{9.3}$$

and, in fitting this solution to experimental observations, only two constants, $k_1$ and $L_1$, have to be determined. Kirkwood and Brinkley do not discuss the physical significance of their length $L_1$, but fit the solution (9.1), (9.2), (9.3) asymptotically to the numerical solution of their equations closer in, and thus derive definite values for $k_1$ and $L_1$. In fact, when written in the form

$$\frac{P - p_0}{p_0} = \frac{(k_1L_1)(1 + \lambda^2)}{R\,(\log R - \log L_1)^{\frac{1}{2}}}, \tag{9.4}$$

$$\frac{I\,a_0}{p_0} = \frac{(k_1L_1)^2(1 + \lambda^2)}{2R}, \tag{9.5}$$

$$\tau\,a_0 = (k_1L_1)(\log R - \log L_1)^{\frac{1}{2}}, \tag{9.6}$$

it is clear that this solution can be used to determine definite values of $k_1$ and $L_1$ by fitting to experimental results or calculations, and thus the form of the solution (9.4) to (9.6) involves inherently some definition of a particular $L_1$, different from the precise definition of L used in the hypothesis of section 6.

Since equation (9.5) is identical with (8.1), the fitting of this solution to the same set of observations should give the same value for $k_1L_1$ as obtained in section 8 for $kL$.

For comparison, some values of $k_1L_1$ obtained by Kirkwood and Brinkley, both by choosing the initial data of their solution to fit experimental data, and by choosing initial data from the thermodynamic properties of the explosive, are quoted here.

| | | |
|---|---|---|
| <u>Reference 2.</u> | 1 lb. Cast Pentolite | |
| | From peak overpressure solution | $k_1^2L_1^2 = 8.73$ ft.$^2$ |
| | | ($L_1 = 8.09$ ft., $R_1 = 0.365$) |
| | From positive impulse solution | $k_1^2L_1^2 = 8.53$ |
| <u>Reference 2.</u> | 1 lb. Torpex II | |
| | From peak overpressure solution | $k_1^2L_1^2 = 9.21$ ft.$^2$ |
| | | ($L_1 = 8.32$ ft., $k_1 = 0.365$) |
| | From positive impulse solution | $k_1^2L_1^2 = 8.85$ ft.$^2$ |
| <u>Reference 17.</u> | 1 lb. TNT | |
| | From positive impulse solution | $k_1^2L_1^2 = 7.97$ |
| | From positive overpressure solution | $k_1^2L_1^2 = 7.96$ |
| | | ($L_1 = 8.41$ ft., $k_1 = 0.335$) |
| <u>Reference 18.</u> | 1 lb. TNT | |
| | From positive impulse solution | $k_1^2L_1^2 = 7.50$ |
| | From positive overpressure solution | $k_1^2L_1^2 = 7.49$ |
| | | ($L_1 = 8.14$ ft., $k_1 = 0.336$) |

(c) It has frequently been suggested that a blast wave may ultimately tend to the form of an N-wave with a second shock about equal in strength to the leading shock. The development given here suggests, on the other hand, that no conclusions can be drawn about the air motion behind the limiting characteristic, on the strength of the governing equations and leading boundary conditions only. Experiment [5,6] shows that the shock corresponding to the second leg of the N is consistently very much weaker than the leading shock. In the case of a conventional explosion, moreover, the second leg of the N is the third shock, and the shock next behind the leading shock is roughly in the middle of the sloping part of the N. It is this second shock which [cf. 7,8] originates at the surface of the explosive and is reflected outwards again after implosion at the centre. This type of shock is peculiar to conventional explosives, and consequent upon the infinite pressure gradient immediately behind a spherical detonation wave.

It is the author's opinion that secondary shocks are not essential to the ultimate forms of blast waves, and that their presence or absence is dependent on the nature of the explosion, or, more precisely, on the motion of the 'explosive products' which act like a piston in causing the explosion. As remarked above, the second shock from a conventional explosive is peculiar to this type of explosive. In atomic explosions no secondary shock is observed at all [15, page 50, footnote]. The author believes that, if an explosion were caused by a spherical piston which returned to its initial position, a true N-wave would ultimately form, just as, in the corresponding case of a <u>finite</u> body in steady supersonic flow, the bow and tail shocks ultimately form an N-wave far out from the body. In this latter case, the shape of the body determines the 'piston curve', which, when the body is finite, returns to its initial position.

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## Table 8.1

1 lb. RDX/TNT at sea-level. $p_o = 14.7$ lb.wt./ins.$^2$ $a_o = 1116$ ft./sec.

| R (ft.) | ($\underline{P}$ - 1) | I ($\frac{\text{lb.wt.msecs.}}{\text{ins.}^2}$) | $\tau$ (msecs.) | $\tau_x$ (msecs.) | $\frac{2\,I\,R\,a_o}{p_o(1+\lambda^2)}$ (ft.$^2$) | $\frac{(\underline{P}-1)\tau_x R\, a_o(\sqrt{3}+1)}{(1+\lambda^2)\sqrt{3}}$ (ft.$^2$) | L (ft.) | L = 3.8 ft. k = 0.716 $k^2L^2 = 7.4$ | | | $\frac{(P-p_o)\tau_x}{(3-\sqrt{3})\,I}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | 1(P) | 1($\tau$) | 1($\tau_x$) | |
| 4 | 3.50 | 12.0 | .80 | .22 | 6.25 | 4.65 | 2.69 | | | | .74 |
| 5 | 2.09 | 10.0 | 1.10 | .35 | 6.51 | 5.52 | 2.90 | | | | .85 |
| 6 | 1.40 | 8.8 | 1.35 | .475 | 6.87 | 6.02 | 3.04 | | | | .88 |
| 8 | 0.78 | 6.9 | 1.74 | .69 | 7.18 | 6.50 | 3.16 | | | | .90 |
| 10 | 0.511 | 5.70 | 2.03 | .87 | 7.42 | 6.71 | 3.23 | | | | .90 |
| 15 | 0.256 | 3.85 | 2.51 | 1.20 | 7.52 | 6.95 | 3.36 | | | | .93 |
| 20 | 0.165 | 2.82 | 2.82 | 1.42 | 7.34 | 7.07 | 3.43 | .22 | .20 | .19 | .96 |
| 25 | 0.120 | 2.23 | 3.04 | 1.575 | 7.26 | 7.13 | 3.47 | | | | .98 |
| 30 | 0.094 | 1.86 | 3.21 | 1.70 | 7.26 | 7.23 | 3.59 | .28 | .26 | .24 | 1.00 |
| 35 | 0.077 | 1.60 | 3.35 | 1.81 | 7.29 | 7.36 | 3.77 | | | | 1.01 |
| 40 | 0.065 | 1.42 | 3.46 | 1.89 | 7.39 | 7.41 | 3.88 | .34 | .30 | .27 | 1.00 |
| 50 | 0.050 | 1.18 | 3.65 | 2.03 | 7.68 | 7.66 | 4.23 | .42 | .32 | .29 | 1.00 |
| | (0.0485) | (1.14) | " | " | (7.42) | (7.43) | (3.92) | (.36) | " | " | (1.00) |
| (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) | (11) | (12) |

## Table 8.2

1 lb. TNT at sea-level. $\quad p_0 = 14.78$ lb.wt./ins.$^2$ $\quad a_0 = 1119.6$ ft./sec.

| R | $(\underline{P} - 1)$ | I | $\tau$ | $\tau_x$ | $\frac{2 I R a_0}{p_0(1+\lambda^2)}$ | $\frac{(\underline{P}-1)\tau_x R a_0(\sqrt{3}+1)}{(1+\lambda^2)\sqrt{3}}$ | L | L = 4.5 ft. k = 0.6025 $k^2L^2 = 7.35$ ft$^2$ | | | $\frac{(P-p_0)\tau_x}{(3-\sqrt{3}) I}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| (ft.) | | $\frac{\text{lb.wt.msecs.}}{\text{ins.}^2}$ | msecs. | msecs. | (ft$^2$) | (ft$^2$) | ft. | $1(P)$ | $1(\tau)$ | $1(\tau_x)$ | |
| 4 | 3.15 | 10.5 | .88 | .24 | 5.45 | 4.58 | 2.44 | | | | .84 |
| 5 | 1.88 | 9.1 | 1.13 | .36 | 5.90 | 5.12 | 2.69 | | | | .87 |
| 6 | 1.26 | 8.0 | 1.34 | .475 | 6.23 | 5.44 | 2.86 | | | | .87 |
| 8 | .71 | 6.4 | 1.70 | .70 | 6.64 | 6.02 | 3.17 | | | | .91 |
| 10 | .47 | 5.3 | 1.97 | .88 | 6.88 | 6.26 | 3.32 | | | | .91 |
| 15 | .241 | 3.6 | 2.42 | 1.20 | 7.01 | 6.57 | 3.59 | | | | .94 |
| 20 | .158 | 2.75 | 2.72 | 1.425 | 7.14 | 6.82 | 3.86 | .10 | .11 | .11 | .95 |
| 25 | .117 | 2.25 | 2.93 | 1.59 | 7.30 | 7.04 | 4.19 | | | | .96 |
| 30 | .093 | 1.90 | 3.11 | 1.72 | 7.40 | 7.26 | 4.43 | .155 | .15 | .135 | .98 |
| 35 | .077 | 1.68 | 3.26 | 1.825 | 7.63 | 7.45 | 4.61 | | | | .975 |
| | (.076) | (1.62) | " | " | (7.35) | (7.35) | (4.49) | | | | (1.00) |
| 40 | .065 | 1.50 | 3.38 | 1.91 | 7.78 | 7.52 | 4.72 | .21 | .16 | .15 | .965 |
| | (.0635) | (1.42) | " | " | (7.37) | (7.35) | (4.49) | (.18) | " | " | (1.00) |
| 50 | .0505 | 1.25 | 3.59 | 2.05 | 8.11 | 7.84 | 5.07 | .28 | .16 | .16 | .97 |
| | (.0475) | (1.13) | " | " | (7.33) | (7.37) | (4.33) | (.19) | " | " | (1.00) |
| (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) | (11) | (12) |

[redacted]



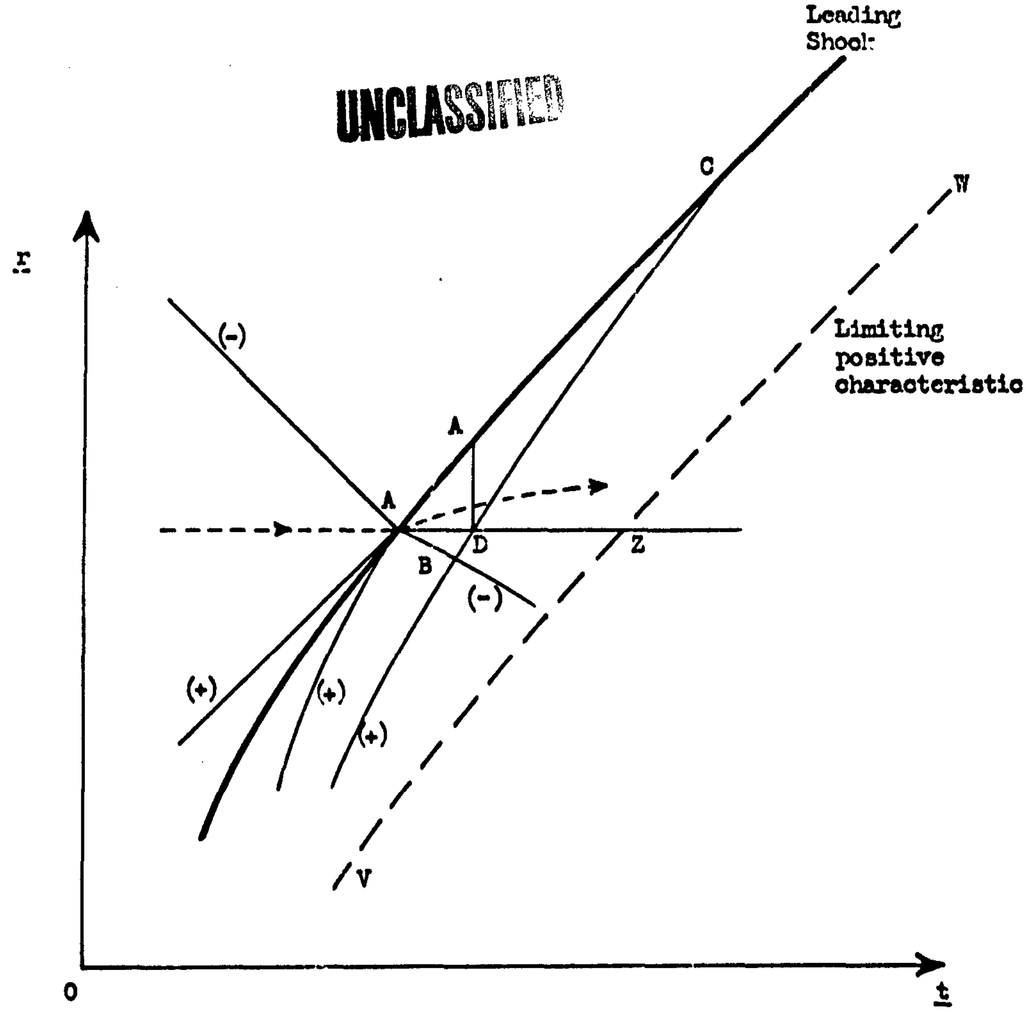


Figure 1.



[redacted]

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[redacted]

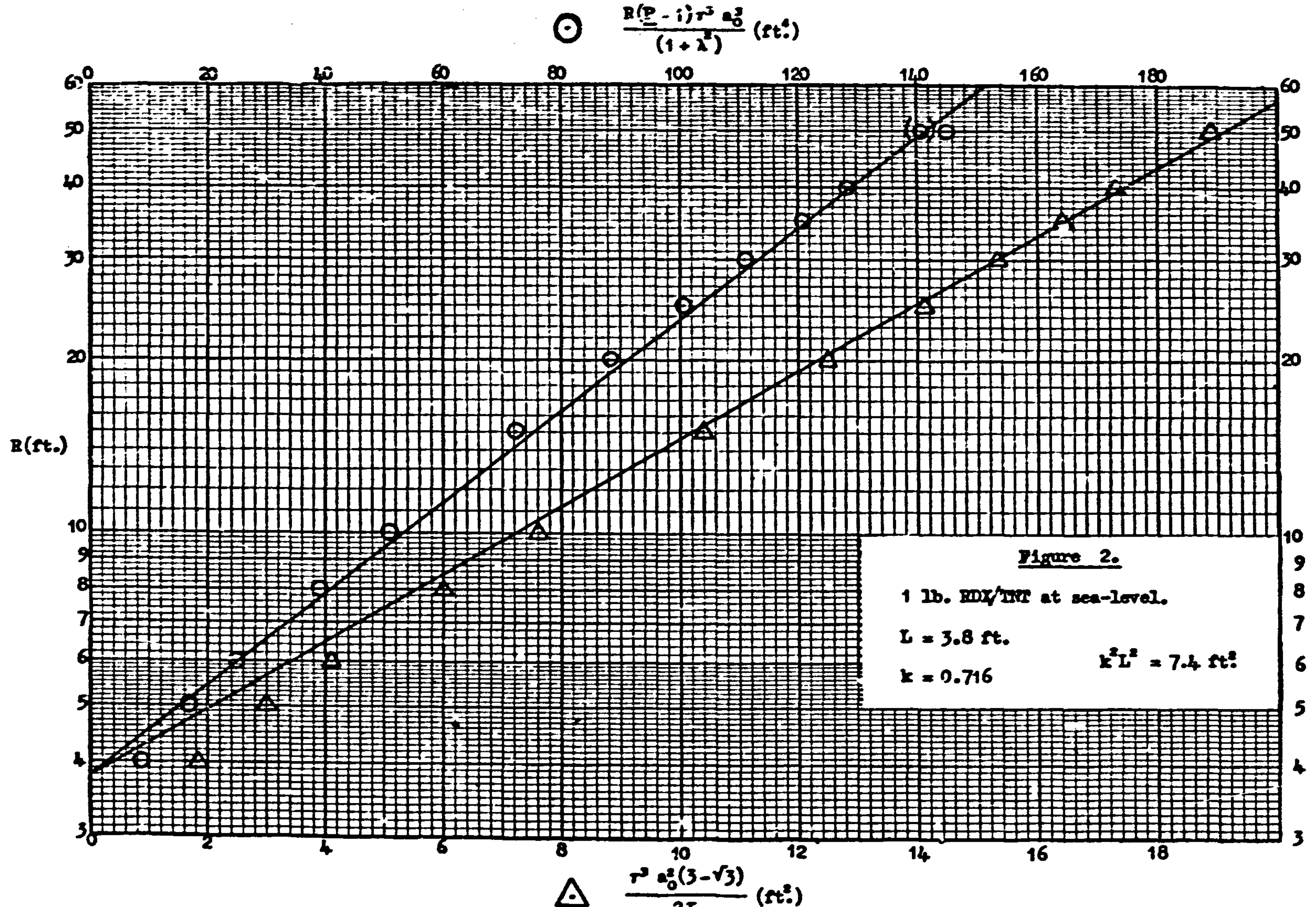


**Figure 2.**

1 lb. RDX/TNT at sea-level.

$L = 3.8$ ft.

$k = 0.716$

$k^2L^2 = 7.4$ ft.²

[redacted]

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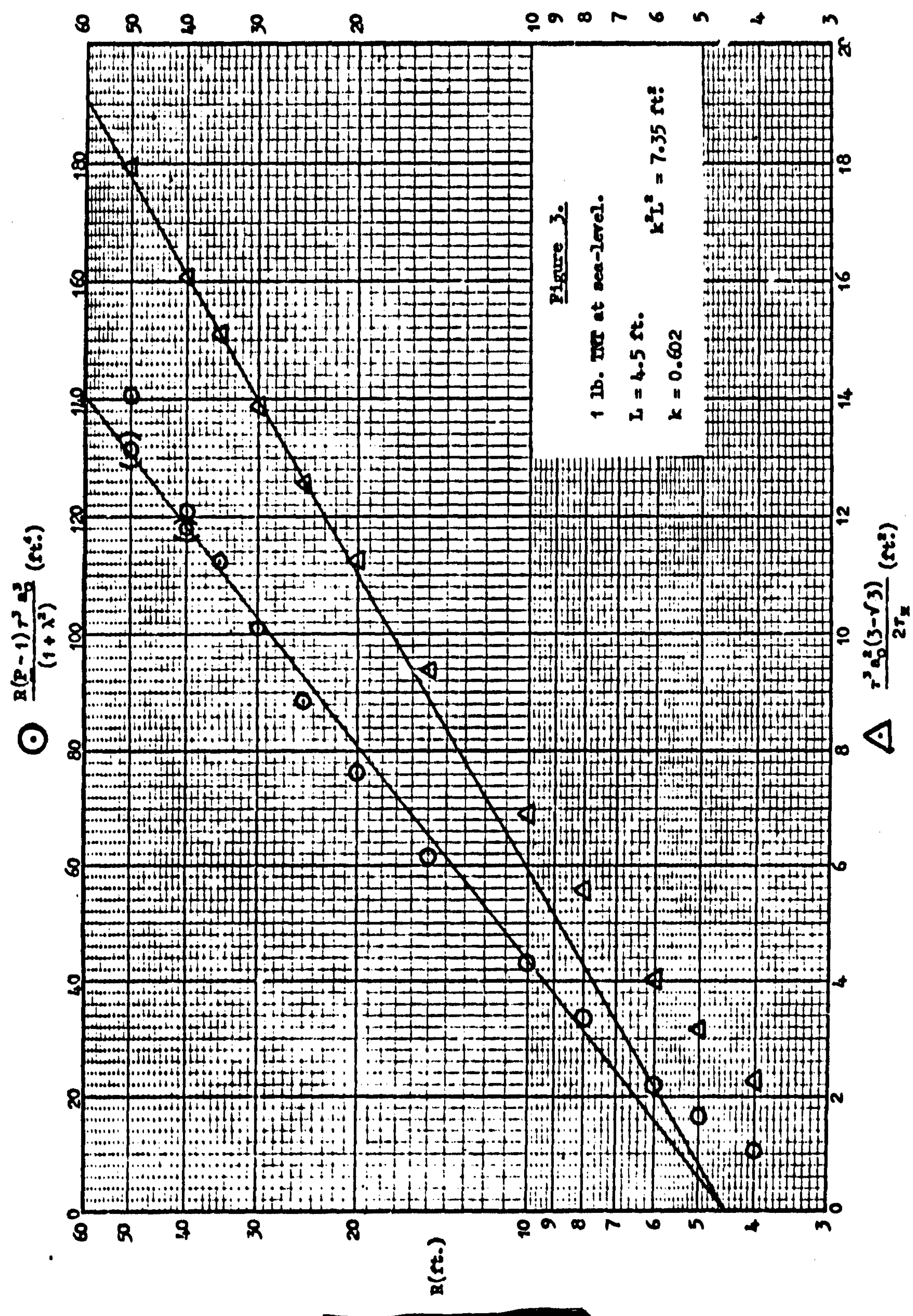
[redacted]

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Information Centre
Knowledge Services
[dstl] Porton Down,
Salisbury
Wilts
SP4 0JQ
22060-6218
Tel: 01980-613753
Fax 01980-613970

Defense Technical Information Center (DTIC)
8725 John J. Kingman Road, Suit 0944
Fort Belvoir, VA 22060-6218
U.S.A.

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